Definition of the equivalent atmospheric stability for wind turbine fatigue load assessment

M C Holtslag, W A A M Bierbooms and G J W van Bussel
Kluyverweg 1, 2629 HS Delft, The Netherlands
E-mail: m.c.holtslag@tudelft.nl

Abstract. In this paper the dependence of wind turbine fatigue loads on atmospheric stability is assessed. It is shown that fatigue loads depend strongly on stability, and highest loads occur for very unstable conditions. For a given hub height wind speed one can define an equivalent atmospheric stability that corresponds to the same cumulative loads as if one performs an infinite amount of simulations for all stability conditions that may occur. It is shown that stability, conditionalised to hub height wind speed, is approximately normally distributed and the equivalent stability corresponds well to the mean stability for a given hub height wind speed. If one follows the IEC guidelines for offshore sites, neglecting atmospheric stability, one will compute higher cumulative lifetime fatigue loads ($\approx 10\%$). This overestimation is caused by conservatism in both wind shear and turbulence levels, which is explicitly shown for the turbulence levels analyzed in this paper.

1. Introduction
Recent studies have shown the significance of atmospheric stability on wind turbine power production [1] as well as on wind turbine loads [2]. This is directly caused by the underlying atmospheric physics which show that atmospheric stability influences both wind shear and turbulence properties [3]. Due to this dependence, it is common procedure to adopt stability classes in wind energy research since this limits the amount of necessary simulations and thus the computational cost of the required simulations. Let us define stability in terms of the Obukhov length $L$

$$L = - \frac{u^3}{\kappa \frac{g}{\theta_v} \bar{w} \bar{\theta}'}$$

(1)

where $u_*$ is the friction velocity, $\kappa$ is the von Karman constant, $g/\theta_v$ is the buoyancy parameter and $\bar{w} \bar{\theta}'$ is the turbulent flux of virtual potential heat at the surface. The Obukhov length $L$ is a continuous parameter that ranges from $-\infty$ to $+\infty$, and the choice of stability class boundaries in research is not physically based but arbitrarily chosen. This raises the question if the procedure to define stability classes is valid. For the turbulence intensity in fact a similar problem arises, though one typically chooses one characteristic turbulence intensity for wind turbine fatigue load assessment. It can be shown that given the lognormal distribution of the turbulence intensity, and the linear dependence of fatigue loads on turbulence intensity, a so-called equivalent turbulence is appropriate for fatigue load assessment [4]. In this study the dependence of fatigue loads on stability is assessed, and it is aimed to define an equivalent stability similar as is often used for turbulence.
2. Atmospheric Conditions

The lowest 90m of the marine atmospheric boundary layer is well described by Monin-Obukhov similarity theory [5, 6], and in this study we adopt Monin-Obukhov Theory to describe atmospheric conditions as a function of stability. All equations shown in this section originate from previous research (currently under submission for publication) based on one year of observation data taken from a metmast sited 85 km offshore in the Dutch Northsea area. Besides, also the guidelines proposed by the IEC are mentioned [7]. Unless stated otherwise, all equations and coefficients shown in this section originate from previous research that was mentioned.

For wind shear it has been shown that the stability corrected logarithmic wind shear profile performs well, though one has to be careful to select appropriate stability correction functions. In this study we adopt the stability correction function of Holtslag [8] for stable condition, and the function that follows the free convection limit for unstable conditions [9]. Combined this leads to the following set of equations

\[
U(z) = \frac{u_*}{\kappa} \left[ \ln \left( \frac{z}{z_0} \right) - \Psi(\zeta) \right]
\]

(2)

\[
\Psi(\zeta \leq 0) = 1.5 \ln \left( \frac{1 + x + x^2}{3} \right) - \sqrt{3} \arctan \left( \frac{2x + 1}{\sqrt{3}} \right) + \frac{\pi}{\sqrt{3}}
\]

(3)

\[
\Psi(\zeta \geq 0) = -1.67\zeta + 9.52e^{-0.35\zeta} - 9.52
\]

(4)

\[
x = (1 - 12.87\zeta)^{1/3}
\]

(5)

An explanation of symbols can be found in [3], but note that \( \zeta = z/L \) is the non-dimensional stability parameter, and \( L \) is the Obukhov length. When \( \zeta < 0 \) the atmosphere is unstable, and for \( \zeta > 0 \) the atmosphere is stable. The aerodynamic roughness length \( z_0 \) is parameterised with Charnock’s equation

\[
z_0 = a_c \frac{u_*^2}{g}
\]

(6)

where Charnock’s constant \( a_c \) is assumed to be 0.012 [10]. The above equations can iteratively be solved if one knows \( U \) and \( \zeta \). The IEC guidelines neglect the impact of stability, and typically prescribe the use of either a power law or the neutral logarithmic shear profile. Here we follow the commonly used convention to consider IEC guidelines with a power law

\[
\frac{U_2}{U_1} = \left( \frac{z_2}{z_1} \right)^\alpha
\]

(7)

with \( \alpha = 0.14 \) for offshore sites.

The turbulence intensity (TI) equals the standard deviation of the wind divided by the mean horizontal wind speed. Previous research showed that the standard deviation of the three components of the wind are all well described by the following general equations

\[
\sigma_x(\zeta \leq 0) = A_x u_* (1 - B_x \zeta)^{1/3}
\]

(8)

\[
\sigma_x(\zeta \geq 0) = A_x u_* (1 + C_x \zeta)^{-1/2}
\]

(9)

where \( x = u, v, w \) and earlier research showed \( A_u = 1.99, B_u = 0.33 \) and \( C_u = 1.32 \). In combination with the stability corrected wind shear profile, the TI can be written as

\[
TI(\zeta \leq 0) = \frac{\kappa A_x (1 - B_x \zeta)^{1/3}}{\ln \left( \frac{z}{z_0} \right) - \Psi(\zeta)}
\]

(10)

\[
TI(\zeta \geq 0) = \frac{\kappa A_x (1 + C_x \zeta)^{-1/2}}{\ln \left( \frac{z}{z_0} \right) - \Psi(\zeta)}
\]

(11)
The IEC guidelines also neglect the impact of stability on the TI and prescribe the mean standard deviation of the longitudinal wind component offshore as a function of the surface roughness as calculated with Charnock’s equation

\[
\sigma_u = \frac{U}{\ln \left( \frac{z}{z_0} \right)}
\] (12)

And for simulations the 90% percentile is considered, which equals

\[
\sigma_u = \frac{U}{\ln \left( \frac{z}{z_0} \right)} + 1.84 \times I_{ref}
\] (13)

Where \( I_{ref} \) is the mean TI found from observations at \( U = 15 \) m/s and the coefficient 1.84 has units m/s. For the mean TI one thus finds

\[
TI = \frac{1}{\ln \left( \frac{z}{z_0} \right)}
\] (14)

and for the simulations the 90% percentile is considered

\[
TI = \frac{1}{\ln \left( \frac{z}{z_0} \right)} + 1.84 \frac{I_{ref}}{U}
\] (15)

For the lateral and vertical components we consider here the minimum requirement of the IEC, corresponding to respectively 0.7 and 0.5 times the longitudinal turbulence intensity.

For the turbulence spectrum the one-sided Kaimal spectrum was validated for the offshore environment, for which the general expression used equals

\[
\frac{nS_x(n)}{\sigma_x^2} = \frac{0.164 \left( f/f_0 \right)}{1 + 0.164 \left( f/f_0 \right)^{5/3}}
\] (16)

Here the coefficients of [11] are considered, in which one can also find an explanation of symbols. The stability dependence of the turbulence spectrum is included implicitly in the scaling frequency \( f_0 \). Previous research showed that in general the scaling frequency for the three wind component turbulence spectra follows the following equations reasonably well

\[
f_0 (\zeta \leq 0) = D_x (1 - E_x \zeta)^{-1/2}
\] (17)

\[
f_0 (\zeta \geq 0) = D_x + F_x \zeta
\] (18)

where again \( x = u, v, w \) and we found \( D_u = 0.01, E_u = 0 \) (no stability dependence for unstable conditions) and \( F_u = 0.03 \). The IEC guidelines propose a variety of spectra, but for sake of comparison we consider the proposed Kaimal spectrum for neutral conditions of the IEC.

\[
\frac{nS_x(n)}{\sigma_x^2} = \frac{4 \left( nL_k/U \right)}{(1 + 6nL_k/U)^{5/3}}
\] (19)

here \( L_k \) is a wind component integral scale parameter, which for heights above 60 m equals 340 m.

The summary of equations presented here define the general atmospheric conditions that are considered in this study. The emphasize from here on is to assess if considering atmospheric stability as prescribed with these equations will influence the simulated fatigue loads of a wind turbine operating offshore.
Figure 1. a) Probability density function of stability (85/L) for various wind speeds and b) the distribution parameters as a function of wind speed. The distributions on the left side for 10 and 14 m/s are shifted vertically by a factor 10 and 100 respectively for clarity reasons.

3. Equivalent loads and simulations

In this study we focus on the fatigue damage that will occur at the blade root. We quantify the fatigue damage in terms of the lifetime fatigue damage equivalent load for comparison purposes [4]. The lifetime cumulative equivalent load can in general be determined as

\[ F_{EQ-TOT} = \int_{U_{in}}^{U_{out}} F_{EQ-U}(\overline{U}) \, P(\overline{U}) \, d\overline{U} \]  (20)

Here \( F_{EQ-U} \) is the equivalent load as a function of the mean wind speed, \( P(\overline{U}) \) is the probability density function of wind speed (the Weibull distribution) and \( U_{in} \) and \( U_{out} \) are respectively the start and stop wind speeds of the wind turbine. For a given wind speed, atmospheric stability conditions will vary potentially from extreme unstable to extremely stable (-\( \infty \) to +\( \infty \)). The total equivalent load that occurs for a given wind speed is thus a function of the probability density function of stability at that given wind speed

\[ F_{EQ-U} = \int_{-\infty}^{\infty} F_{EQ-\zeta}(\zeta|\overline{U}) \, P(\zeta|\overline{U}) \, d\zeta \]  (21)

Here \( F_{EQ-\zeta} \) is the fatigue load at a given wind speed as a function of stability, and \( P(\zeta|\overline{U}) \) is the conditional probability density function of stability at a given wind speed (figure 1). Based on the observation data also used to define the atmospheric conditions, it is found that for a given wind speed stability is approximately normally distributed. The dependence of the distribution parameters (that is, the mean and standard deviation of 85/L) on wind speed shows that for increasing wind speeds the atmosphere becomes neutrally stratified with little variation. Note that the 85 here is chosen since the assessment of turbulence conditions is also carried out at 85 meter height (this is the only height where sonic anemometers are installed in the metmast). For weak wind speeds the atmosphere is predominantly unstable stratified, and the spread in stability conditions is significantly larger.
If we approximate the integrals with summations, then the lifetime cumulative equivalent load equals

\[ F_{EQ\text{-TOT}} = \sum_U \left( \sum_\zeta F_{EQ} \left( \zeta | \mathcal{T} \right) P \left( \zeta | \mathcal{T} \right) \Delta \zeta \right) P \left( \mathcal{U} \right) \Delta \mathcal{U} \]  \hspace{1cm} (22)

The equivalent load for a given atmospheric condition (that is, wind speed and stability) can be calculated as

\[ F_{EQ} = \left( \sum F_i^n n_i \right)^{1/m} \]  \hspace{1cm} (23)

here \( F_{EQ} \) is the equivalent load, \( n_{EQ} \) is the number of equivalent cycles, \( F_i \) and \( n_i \) are the actual fatigue load and fatigue cycles and \( m \) is the Whöler exponent. In this study we consider \( n_{EQ} = 10^7 \), and \( F_{EQ} \) is obtained from the simulation software used based on a rainflow counting algorithm.

From equation (22) it is clear that one will perform individual load simulations for all possible hub height wind speeds. It is recognized that for a given hub height wind speed, the TI (or in fact the standard deviation of the wind) will vary. The variability of the TI in load assessment is considered either by the equivalent turbulence, which is done in this study for the stability dependant simulations, or the 90% percentile if one follows the IEC guidelines. The equivalent turbulence assumes that for a given wind speed the standard deviation of the wind is lognormally distributed, which is in practice approximately true. For a lognormally distributed variable \( x \), the lognormal distribution parameters \( M \) and \( S \) are a function of the mean \( \mu_x \) and standard deviation \( \sigma_x \) of \( x \) as

\[ F(x) = \Phi \left( \frac{\ln(x) - M}{S} \right) \]  \hspace{1cm} (24)

\[ M = \ln \left( \frac{\mu_x^2}{\sqrt{\mu_x^4 + \sigma_x^4}} \right) \]  \hspace{1cm} (25)

\[ S = \sqrt{\ln \left( \frac{\sigma_x^2}{\mu_x^2} + 1 \right)} \]  \hspace{1cm} (26)

**Figure 2.** Mean and standard deviation of \( \sigma_u/u_\ast \) as a function of stability for a) unstable and b) stable conditions. Dashed lines correspond to equations (8), (9) and (28) with \( \epsilon = 0.23 \).
Here $\Phi$ is the cumulative distribution function of the standard normal distribution and $F$ is the cumulative distribution function. Hence if we know that the standard deviation of the wind $\sigma_u$ is lognormally distributed, and we know the mean and standard deviation of $\sigma_u$, the distribution parameters $M$ and $S$ are easily calculated. The equivalent turbulence is a function of these distribution parameters \[ \sigma_{EQ} = e^{M+0.5mS^2} \] (27)

Figure 2 shows the mean and standard deviation of the non-dimensional standard deviation of the longitudinal wind component. Due to the normalization all wind speeds are included. We emphasize here on the longitudinal wind component which is most important for the blade root bending moment, though results for the lateral and vertical wind component are similar. It can be seen that in general both the mean and standard deviation of $\sigma_x$ have a similar dependence on stability for all unstable conditions, and for stable conditions for $85/L \leq 1$. We therefore assume the following relation between the mean and standard deviation of $\sigma_x$

$$\sigma_{\sigma_x} = \epsilon \mu_{\sigma_x}$$

Where $\epsilon$ is a constant of approximately 0.2 to 0.25. The equivalent turbulence can then be written as

$$\sigma_{EQ} = \mu_{\sigma_x} \left( \epsilon^2 + 1 \right)^{0.5m-0.5}$$

And we note that for strong stable conditions $\sigma_{EQ}$ is underestimated based on figure 2. Since $\mu_{\sigma_x}$ has been specified earlier (figure 2), one only has to choose a valid value for $m$ depending on wind turbine specifications.

As mentioned, the fatigue loads will be calculated for the blade root bending moment with design simulation software. We consider the 5MW NREL reference wind turbine [12], and the design software Bladed is used for all simulations. The 5MW NREL turbine has a 90 m hub height, and a 63 m radius with a cut-in wind speed of 3 m/s and a cut-out wind speed of 25 m/s. It is expected that the difference between $z_{hub}/L$ and $85/L$ in terms of the stability distribution is negligible. Although we consider the offshore environment, the focus is purely on offshore atmospheric conditions hence to keep results as clear as possible waves and currents are not included in the simulations. For each considered atmospheric condition (wind speed and stability) 6 turbulence seeds are included in the analyses to reduce statistical uncertainty, and simulations are run for 600 seconds. The Whöler exponent is assumed to be 12 for the blade root [2]. Wind speed increments of 1 m/s are considered for $\Delta U$, and stability increments of 1 are considered between $-10 < \zeta < 5$ with an increase in density to 0.1 between $-1 < \zeta < 1$ due to rapid changes in shear and turbulence levels for near neutral conditions. Although in principle one would prefer to perform analyses for $-\infty < \zeta < \infty$, this is infeasible and the above mentioned procedure results in including at least 95% of all situations that will occur in reality.

4. Results

The dependence of wind turbine fatigue loads on atmospheric stability is shown in a series of substeps in figure 3. First the dependence of $F_{EQ-\zeta}$ on stability is assessed, and the contribution of the probability density function of atmospheric stability is considered separately. Next the dependence of $F_{EQ-U}$ on wind speed is analysed, and the impact of the probability density function of wind speed is considered separately.

With respect to $F_{EQ-\zeta}$, it is clear that fatigue loads are highest for strong unstable conditions caused by a strong increase in turbulence levels (figure 3a). For very stable conditions fatigue loads are lowest, which might be counterintuitive due to the strong wind shear levels typically observed during such conditions. The decrease of $F_{EQ-\zeta}$ is strongest when the atmosphere
shifts from neutral to weakly stable conditions, which is caused by a strong decrease in atmospheric turbulence. When the stability changes from weakly stable conditions to strongly stable conditions the fatigue loads hardly change. For such changing conditions both wind shear increases and turbulence levels decrease, and there is apparently a balance in terms of the impact on fatigue loads since $F_{EQ-\zeta}$ is hardly changing. A comparison with the results obtained following the IEC guidelines shows that for low wind speeds (here shown for $U = 7$ m/s) the fatigue loads correspond to strong unstable conditions. When wind speeds increase the calculated fatigue loads correspond to weak unstable and eventually to near neutral conditions.

When $F_{EQ-\zeta}$ is multiplied with the normal distribution of atmospheric stability one can see that for a given wind speed the fatigue loads are nearly normally distributed as well (figure 3b). This shows that extreme stability conditions are unimportant for fatigue load assessment, despite the occurrence of strong wind shear (stable) or high turbulence levels (unstable). For a given wind speed, one might thus expect that the mean of the stability distribution is a good estimate to incorporate into the load assessment instead of performing simulations for all possible values.
Figure 4. a) Determination of the equivalent stability $\zeta_{EQ}$ as the intersect between $F_{EQ-\tau}$ and $F_{EQ-\zeta}$, and b) the dependence of $\zeta_{EQ}$ on wind speed compared to the mean stability for each wind speed.

Once the summation over stability is performed one can compare the equivalent loads as a function of wind speed including stability to those calculated based on the IEC guidelines (figure 3c). Here it can be seen that for the majority of wind speeds loads are overestimated by the IEC guidelines. Only for very low wind speeds, where fatigue loads are low anyway, the guidelines correspond well to the stability dependant simulations. The overestimation of the IEC guidelines can be explained by the fact that if one neglects atmospheric stability, one has to be (slightly) conservative on both wind shear and turbulence levels. In reality however, the occurrence of strong wind shear and high turbulence levels never occurs, since for unstable conditions there is little wind shear while for stable conditions there is little turbulence. As such, including atmospheric stability causes a reduction in the required conservatism.

The relative contribution of $F_{EQ-\tau}$ at each wind speed to the total cumulative equivalent load depends on the Weibull distribution. It can be seen that wind speeds around rated (8 to 14 m/s) contribute most to the total equivalent loads (figure 3d). One can see here as well that if one follows the IEC guidelines, the overestimation of the equivalent loads primarily occurs for these wind speeds. Since high wind speeds do not occur often, the overestimation of equivalent loads by the guidelines for these strong winds is unimportant in terms of estimated lifetime equivalent loads. After summation over wind speed of the profiles shown in figure 3d one can compute the lifetime equivalent loads. For the wind speeds considered here, the lifetime cumulative equivalent loads are 10% less if one considered atmospheric stability compared to following the IEC guidelines. Since high wind speeds do not contribute significantly to the cumulative equivalent loads, it is expected that extending the simulations up to 25 m/s would cause little deviation in this difference percentage.

The results obtained here including atmospheric stability require a large number of simulations. Based on figure 3, one can compute an equivalent stability value for which one will obtain exactly the same $F_{EQ-\tau}$ as if one would perform an infinite amount of simulations with all possible values of $L$. This equals the intersect between $F_{EQ-\zeta}$ and $F_{EQ-\tau}$ for a given wind speed $\bar{U}$ (see also figure 4a). Figure 4b shows the equivalent stability as a function of wind speed and for comparison purposes the mean of the fitted stability distribution (figure 1b) is included as well. It is found that the equivalent stability varies from unstable (low wind
Longitudinal turbulence intensity as a function of wind speed according to observations and guidelines for a) all conditions and b) neutral conditions only. Note the different y-axes limits.

speeds, $\zeta_{eq} = -1.5$) to approximately neutral (high wind speeds, $\zeta_{eq} = 0$). The equivalent stability shows a similar change with wind speed as the mean stability from the fitted normal distribution, and also a similar order of magnitude. As such, a first approximation would be that the mean stability is a good estimate of the equivalent stability if one wants to include atmospheric stability in fatigue load simulations.

5. Discussion

Due to the incorporation of atmospheric stability into the fatigue load simulations, it is evident that one has to be less conservative in the definition of wind shear and turbulence levels since shear and turbulence are inversely dependant on atmospheric stability. It can be shown easily that the shear profile prescribed by the guidelines corresponds to a slight stable atmosphere. For the mean turbulence intensity one can make a similar comparison. If one does not consider atmospheric stability (figure 5a), the guidelines differ slightly from the observations. The typical result however that the turbulence intensity increases with increasing surface roughness is reasonably well captured for wind speeds above 12 m/s, though absolute turbulence levels are slightly overestimated (see also [13]). For low wind speeds one finds a strong deviation however, which can only be explained if one does consider atmospheric stability (for such wind speeds it is found that the atmosphere is predominantly unstable, thus turbulence levels increase).

It would be more fair to only consider neutral conditions, and see if the guidelines correspond well to the average turbulence intensity for neutral conditions (figure 5b). For neutral conditions the guidelines agree well with observations in terms of the wind speed dependence of the turbulence intensity, however there is a strong overestimation of the turbulence intensity. This shows that the guidelines prescribe turbulence levels that are typically found for (slight) unstable conditions.

Combined, the guidelines prescribe shear that correspond to stable conditions (thus fairly high) and turbulence levels that correspond to unstable conditions (also fairly high). Including atmospheric stability prevents the necessary conservatism in both wind shear and turbulence, which decreases simulated equivalent loads by 10% as shown in the results.
6. Conclusions

The impact of atmospheric stability on the fatigue loads of the blade root bending moment is shown based on a series of fatigue load simulations. It has been shown that for a given hub height wind speed, fatigue loads are highest for strong unstable conditions and lowest for strong stable conditions. Besides, there is a rapid change in the fatigue loads when atmospheric stability changes from slightly unstable towards slightly stable conditions. Based on fitted stability distributions, it is shown that the cumulative loads over all stability conditions for a given hub height wind speed increase rapidly with increasing wind speed. If one also considers the Weibull distribution typically found offshore, then one finds that the highest contribution to lifetime fatigue loads comes from conditions where hub height wind speeds are between 8 and 14 m/s. A comparison with IEC guidelines has been carried out, and it is found that if one does not consider stability one will overestimate loads for all wind speeds. This is caused by simultaneous conservatism in both wind shear and turbulence, since guidelines prescribe relatively high wind shear and turbulence levels similarly, while in reality either wind shear or turbulence intensity is relatively low depending on atmospheric stability.

Including atmospheric stability in the load assessment results in a significant increase in the amount of required simulations. The equivalent stability \( \zeta_{EQ} \) has been defined to reduce the computational time required for fatigue load simulations that include atmospheric stability. It is found that \( \zeta_{EQ} \) corresponds quite well to the mean stability for a given hub height wind speed, and for the observation data used in this study \( \zeta_{EQ} \) in terms of \( 85/L \) ranges from -1.5 (unstable) for low wind speeds towards 0 (neutral) for high wind speeds. The equivalent stability allows one to incorporate atmospheric stability into load assessment without increasing the amount of simulations required.

Acknowledgments

This study is financed and has been carried out in scope of the FLOW (Far and Large Offshore Wind) research project.

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